

Comparison of Algebraic Reconstruction Methods in Computed Tomography

G Vijayalakshmi, P Vindhya

Department of Computer Science & Engineering, GKCE-JNTUA
Sullurpet, Andhra Pradesh, India

Abstract— Advances in computed tomography and availability of high performance computing in the last decade have shown importance of iterative reconstruction in combination with statistical methods. An attempt is made to compare two existing iterative methods used for reconstruction namely ART and SART. The results show that ART is a highly computational intensive and time taking process compared to SART for reconstruction. The estimated RRMS error between the reconstructed images by using ART and SART with respect to the original phantom does not show much variation with increase in number of image projections used for reconstruction.

Keywords— Computed Tomography, ART, SART, RRMSE.

I. INTRODUCTION

X-ray Computed Tomography (CT) plays a very important role in the field of medicine and also non-medical usage such as non-destructive testing (NDT) in industry, archaeology, soil science, biology etc. CT refers to developing cross-sectional images of an object of interest from either transmitted or reflected data collected by illuminating the object of interest from many different directions [1].

CT mathematical process includes two major categories namely, analytical reconstruction as well as iterative reconstruction methods. Filtered back-projection (FBP) is a type of analytical reconstruction method that is widely used in clinical CT scanning systems. This method is found to be computational efficient and has numerical stability [2]. As research in low dose CT is emerging iterative reconstruction methods are gaining their importance. Iterative reconstruction reduces image noise significantly without loss of diagnostic information and holds the potential for substantial radiation dose reduction over traditional FBP [3].

Iterative reconstruction methods allow integrating various physical models that can reduce image noise and various artifacts depending on the degree of modelling [4]. Modelling the causes of artefacts during the reconstruction procedure instead of trying to eliminate, make iterative methods to represent more intuitive way of image reconstruction. Different iterative methods have been developed over years and a comprehensive list has been furnished in [4] can be referred.

In the current paper two iterative or algebraic methods namely Algebraic Reconstruction Technique (ART) and Simultaneous Algebraic Reconstruction Technique (SART) have been compared with varying number of projections generated from a test phantom and the results are presented.

II. MATERIALS AND METHODS

The attenuation of X-rays passing through the object of interest is defined by Beer's law given by equation (1). The equation (1) can be rewritten in the form of equation (2).

$$I = I_0 \exp \left[\sum_i (-\mu_i x_i) \right] \quad (1)$$

$$\ln(I/I_0) = - \sum_i \mu_i x_i \quad (2)$$

Where, I_0 and I are the initial and final X-ray intensities, μ_i is the i^{th} material linear attenuation coefficient, x_i is the length of the X-ray path in the material. Equation (2) takes a general form of algebraic equation denoted by $A \cdot x = b$. Given a projection each row stores information about the slice to be reconstructed and each pixel in a row indicates an attenuated X-ray path through the object. We attain a number of algebraic equations with unknowns and it can be formulated into a matrix notation for solving as given by equation (3).

$$[A][x] = [b] \quad (3)$$

Where $[A]$ denotes weighting factor matrix that shows the contribution of individual cells, $[x]$ represents the image to be reconstructed and $[b]$ represents the projection data measured by detector.

A. ART

The ART algorithm used for reconstruction is defined as follows:

1. Initialize $P = 0$
2. For each iteration of k
3. For each row a_j^T of A
4.
$$P \leftarrow P + \alpha \frac{(b_j - a_j^T P) a_j}{a_j^T a_j}$$

Where, α indicates the relaxation parameter.

B. SART

The SART algorithm used for reconstruction is defined as follows:

1. For each projection
2. For each row i in projection
3.
$$q_i \leftarrow q_i + \alpha \frac{(b_i - a_i^T P) a_i}{a_i^T a_i}$$
4.
$$q_i \leftarrow \frac{q_i}{\sum_j a_{ij}}$$
5.
$$P \leftarrow P + q$$

Image Evaluation

The reconstructed images are evaluated using relative root mean square error (RRMSE) and the square Euclidean distance (sqEuc) [5] defined as follows:

$$RRMSE = \sqrt{\frac{\sum_{i=1}^N (x_i - x_i^{ref})^2}{\sum_{i=1}^N (x_i^{ref})^2}} \tag{4}$$

$$sqEuc = 1 - \frac{1}{N} \sum_{i=1}^N (x_i - x_i^{ref})^2 \tag{5}$$

Where x is the reconstructed image and x^{ref} is the reference image.

C. Test Phantom & Tools

The test phantom used is known as Shepp-Logan phantom widely used by researchers in the field of CT. It represents a head image with the different elliptical shapes and grey scale intensities representing parts of brain as shown in Fig. 1. In the present study an image of size 128×128 pixels is taken for testing. The original image is projected into radon space and then both ART and SART algorithms are independently executed for reconstructing the image.



Fig. 1 Test phantom of size 128×128 pixels

For the present study MATLAB is chosen as the best available software package for programming and Microsoft Excel is used to generate the test results for comparison.

III. RESULTS AND DISCUSSION

A. Relaxation Parameter vs RRMSE

To understand the impact of relaxation parameter (α) on both ART and SART, a varying alpha value between (0, 2) [5] is carried out. The number of iterations was fixed to 50 for both methods; the number of projections was fixed to 60. Fig. 2 shows the variation of RRMS error with respect to α .

The fast convergence of ART depends on choice of relaxation parameter. No much variation is seen in RRMS error in case of ART, but significant variation is seen for SART.

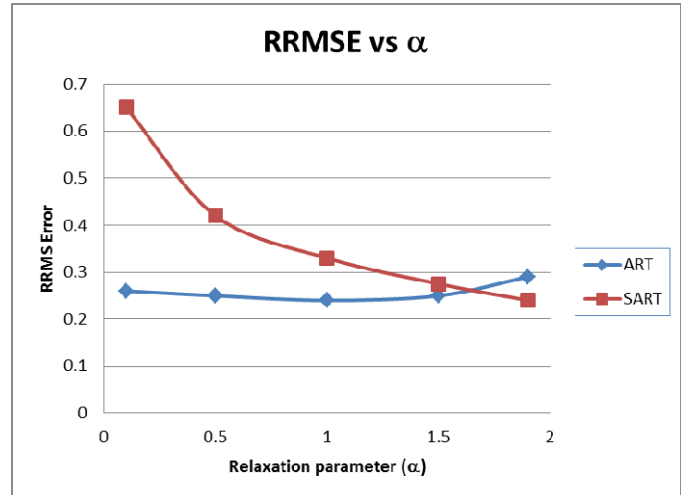


Fig. 2 Variation of RRMSE with relaxation parameter

B. Comparison of ART and SART

After finding the impact of relaxation parameter on both techniques an attempt is made to compare both methods using image evaluation technique defined in earlier section. The relaxation parameter is taken as 1.5 where both techniques are having less difference with respect to RRMS error. The number of iterations is taken as 100 and number of projections is taken as 60. Fig. 3 shows the resulting reconstructed images for the above parameters.

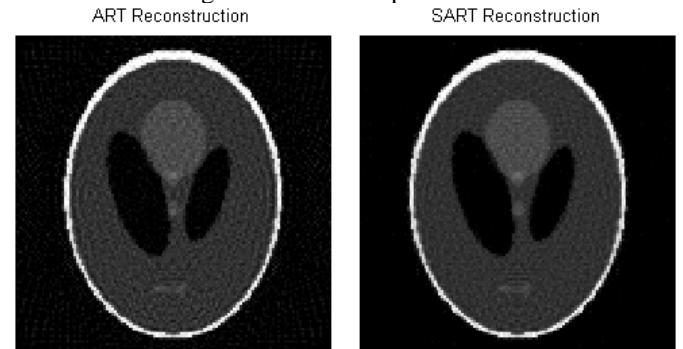


Fig. 3 Both ART and SART reconstructed images ($\alpha=1.5$, No. of projections =60, No. of Iterations = 100)

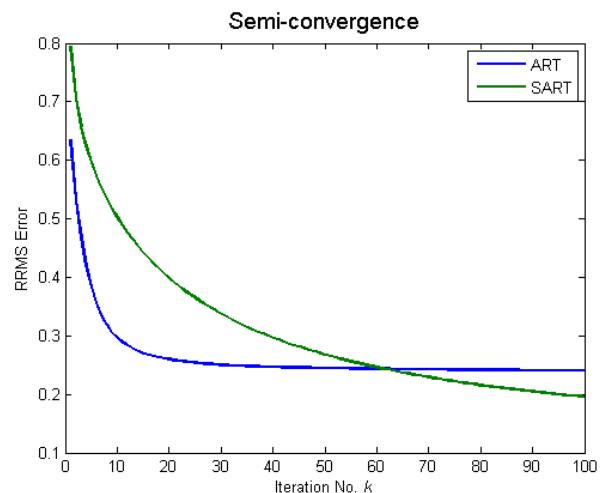


Fig. 4 RRMSE versus No. of Iterations

It can be seen from Fig. 4 that the fast convergence of ART before SART. The RRMSE is stabilized for ART and no significant change is seen as the iterations increase. But in case of SART, the RRMSE decreases as the iterations progress. The sqEuc versus number of iterations as shown in Fig. 5 shows similar pattern with ART attaining a constant value and image quality enhancing in case of SART method.

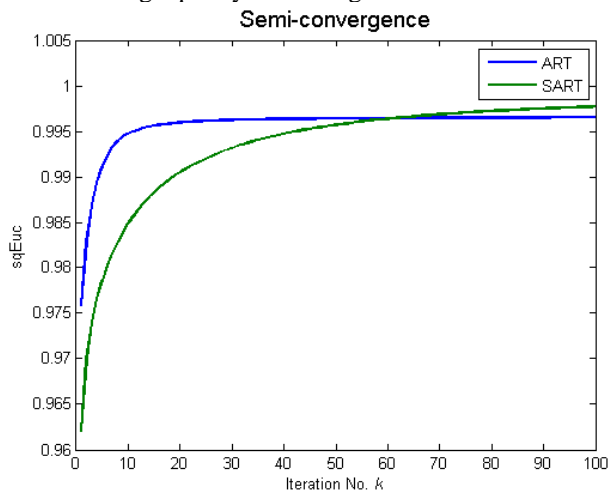


Fig. 5 sqEuc versus No. of Iterations

C. Impact of Projection Number

The impact of number of projections used for reconstruction of image using both ART and SART methods are studied and the results are shown in Table 1. Input parameters used for analysis are relaxation parameter ($\alpha=1.5$) and maximum number of iterations limited to ($k=100$). The results show that as the number projections are decreasing the RRMSE is increasing significantly in case of ART compared to SART method. As the number of projections is decreasing a poorer quality reconstructed image is obtained by using both methods.

TABLE I
PROJECTION NUMBER AND RRMS ERROR

Projection Number	RRMS Error	
	ART	SART
20	0.48	0.29
36	0.35	0.24
60	0.24	0.20
90	0.15	0.17

IV. CONCLUSIONS

The selection of relaxation parameter helps in SART method for better image quality and in the case of ART faster convergence. The number of iterations is playing a significant role in the case of SART rather than ART in enhancing the image quality as iterations progress. The number of projections considered as input for reconstruction also shown an impact on the image quality in the case of ART compared to SART. The estimated RRMS error between the reconstructed images by using ART and SART with respect to the original phantom does not show much variation with increase in number of image projections used for reconstruction. This show that SART can be implemented for better results when lesser projections are available and also targeting the final image quality.

ACKNOWLEDGMENT

We would like to thank our Principal Dr. E Vijayakrishna Rapaka and Management of GKCE for their extended support in this research work.

REFERENCES

- [1] Kak A.C., Slaney M., Principles of Computed Tomographic Imaging. SIAM, 1987.
- [2] Orentlicher G.P., Haug R.H., Digital Technologies in Oral and Maxillofacial Surgery, an issue of atlas of the oral and maxillofacial surgery clinics of North America. Elsevier, March 2012.
- [3] Moscariello A., et. al., "Coronary CT angiography: image quality, diagnostic accuracy, and potential for radiation dose reduction using a novel iterative image reconstruction technique—comparison with traditional filtered back projection", European Radiology (2011), 21:2130-2138.
- [4] Marcel Beister, Daniel Kolditz, Willi A. Kalender, "Iterative reconstruction methods in X-ray CT", Physica Medica (2012) 28, 94-108.
- [5] Joshua Kim, Huaiqun Guan, David Gersten, and Tiezhi Zhang, "Evaluating of Algebraic Iterative Image Reconstruction Methods for Tetrahedron Beam Computed Tomography Systems", International Journal of Biomedical Imaging Volume 2013, Article ID 609704, 14 Pages.
- [6] W. Qiu, T. Pengpan, N. D. Smith, M. Soleimani, "Evaluating iterative algebraic algorithms in terms of convergence and image quality for cone beam CT", Computer Methods and Programs in Biomedicine 109(3): 313-322 (2013).
- [7] <http://wikipedia.org>
- [8] <http://google.co.in>